Student Name:

Student ID:

Spring 2009/2010
(092)

FINAL EXAM

Computer Science Department
Course # : ICS 253
Subject : Discrete Structures I
Instructor : Dr. Samer Arafat
Date : June 15, 2010

Read the following instructions before you start working on test questions:

1. Write CLEARLY and LEGIBLY. Write SPECIFIC and BRIEF answers
2. Write only ONE ANSWER. Solutions that have more than one answer **shall receive ZERO credit.**
3. Solutions that consist of only a final answer **shall receive ZERO credit.**
4. **Write a STEP-WISE SOLUTION. Missing steps in your solution means that you will loose points.**

Problem # 1 (10 points)

Use **Contraposition** to prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$, such that $n, m \in \mathbb{R}$.

Problem #2 (10 points)

Use set builder notation and logical equivalences to show that $A \cap \overline{A} = \phi$

Problem #3 (10 points)
Assuming a domain of only positive integers, develop a simple (non-recursive) formula for the following sequence:
1, 10, 25, 46, …

Problem #4 (10 points)
Let $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$. Use **strong induction** to show that $a_n \leq 3^n$ for all positive integers n.

<u>Problem #5</u> (15 points)

A. (7 points) How many bit strings of length 10 begin with 1101?

B. (8 points) How many bit strings of length 12 have exactly four 1s and none of these 1s are adjacent to each other?

Ans: $\binom{9}{4}$ (is this correct?)

<u>Problem #6</u> (10 points)

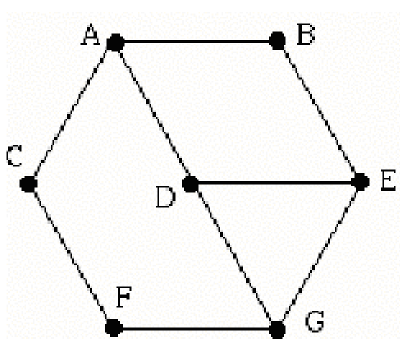
Suppose that you have an unfair (biased) coin, where *p*(heads) = 3/4 and *p*(tails) = 1/4, and supposed that you flip this unfair coin ten times. Find the probability that exactly 9 heads will come up.

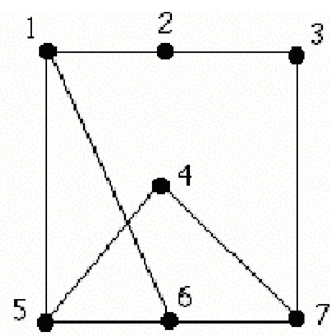
<u>Problem #7</u> (10 points)

Find a recurrence relation for the number of bit strings that contain the string 01.

<u>Problem #8</u> (15 points)

Show whether the following two graphs are isomorphic or not:





<u>Problem #8</u> (10 points)
Represent the following compound proposition using ordered rooted trees, and then write these expressions in postfix notation:

$(\neg p \wedge (q \leftrightarrow \neg p)) \vee \neg q$